Signals, Systems, and Transforms

Fifth Edition

Charles L. Phillips | John M. Parr | Eve A. Riskin

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FIFTH EDITION

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Printer/Binder: *Courier Westford*
Cover Printer: *Courier Westford*

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Library of Congress Cataloging-in-Publication Data
Phillips, Charles L.
Signals, systems, and transforms / Charles L. Phillips, emeritus, Auburn University, Auburn, Alabama, John M. Parr, University of Evansville, Evansville, Indiana, Eve A. Riskin, University of Washington, Seattle, Washington.—Fifth edition.
pages cm.
ISBN-13: 978-0-13-350647-1 (alk. paper)
ISBN-10: 0-13-350647-9 (alk. paper)
1. Signal processing–Mathematical models. 2. Transformations (Mathematics)
3. System analysis. I. Parr, John M. II. Riskin, Eve A. (Eve Ann) III. Title.
TK5102.9.P47 2014
621.382'2—dc23

2013027292

10 9 8 7 6 5 4 3 2 1

PEARSON

ISBN-10: 0-13-350647-9
ISBN-13: 978-0-13-350647-1

To

Taylor, Justin, Jackson, Rebecca, and Alex

Michaela, Cadence, Miriam, and Connor Duncan,

Gary, Noah, and Aden

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PREFACE

The basic structure and philosophy of the previous editions of *Signals, Systems, and Transforms* are retained in the fifth edition. Many end-of-chapter problems have been revised and numerous new problems are provided. Several of these new problems illustrate real-world concepts in digital communications, filtering, and control theory. The end-of-chapter problems are organized so that multiple similar problems are provided. The answer to at least one of each set of similar problems is provided in Appendix H. The intent is to allow students to develop confidence by gaining immediate feedback about their understanding of new material and concepts. All MATLAB examples have been updated to ensure compatibility with the Student Version R2012. We have added the following changes to the fifth edition:

- Presentation of the properties of the Fourier transform is revised and rearranged in Chapter 5.
- A new subsection on the design and analysis of active filters is added in Chapter 6.
- Sampling of continuous-time signals and reconstruction of signals from sample data are now collocated in Chapter 6.
- A new subsection on quantization error is added to Chapter 6.
- A new subsection on system step-response calculation and analysis using the Laplace transform is added to Chapter 7.
- A new subsection on system frequency-response calculation and analysis using the z-transform is added to Chapter 11.
- A new example, showing frequency-response analysis of a finite-impulse-response (FIR) filter using the discrete-time Fourier transform (DTFT), is added in Chapter 12.
- A new example, showing the use of the discrete Fourier transform (DFT) to implement a FIR filter, is added in Chapter 12.
- Several new examples and MATLAB® applications are provided.
- All end-of-chapter problem sets have been revised. The problems for each chapter are now grouped according the applicable section of the chapter.

A companion website at *http://www.ee.washington.edu/class/SST_textbook/textbook.html* contains sample laboratories, lecture notes for Chapters 1–7 and

Chapters 9–12, and the MATLAB files listed in the textbook as well as several additional MATLAB files. It also contains a link to a second website at *http://www.ee.washington.edu/class/235dl/*, which contains interactive versions of the lecture notes for Chapters 1–7. Here, students and professors can find worked-out solutions to all the examples in the lecture notes, as well as animated demonstrations of various concepts including transformations of continuous-time signals, properties of continuous-time systems (including numerous examples on time-invariance), convolution, sampling, and aliasing. Additional examples for discrete-time material will be added as they are developed.

This book is intended to be used primarily as a text for junior-level students in engineering curricula and for self-study by practicing engineers. It is assumed that the reader has had some introduction to signal models, system models, and differential equations (as in, for example, circuits courses and courses in mathematics), and some laboratory work with physical systems.

The authors have attempted to consistently differentiate between signal and system models and physical signals and systems. Although a true understanding of this difference can be acquired only through experience, readers should understand that there are usually significant differences in performance between physical systems and their mathematical models.

We have attempted to relate the mathematical results to physical systems that are familiar to the readers (e.g., the simple pendulum) or physical systems that students can visualize (e.g., a picture in a picture for television). The descriptions of these physical systems, given in Chapter 1, are not complete in any sense of the word; these systems are introduced simply to illustrate practical applications of the mathematical procedures presented.

Generally, practicing engineers must, in some manner, validate their work. To introduce the topic of validation, the results of examples are verified, using different procedures, where practical. Many homework problems require verification of the results. Hence, students become familiar with the process of validating their own work.

The software tool MATLAB is integrated into the text in two ways. First, in appropriate examples, MATLAB programs are provided that will verify the computations. Then, in appropriate homework problems, the student is asked to verify the calculations using MATLAB. This verification should not be difficult because MATLAB programs given in examples similar to the problems are applicable. Hence, another procedure for verification is given. The MATLAB programs given in the examples may be downloaded from *http://www.ee.washington.edu/class/SST_textbook/textbook.html*. Students can alter data statements in these programs to apply them to the end-of-chapter problems. This should minimize programming errors. Hence, another procedure for verification is given. However, all references to MATLAB may be omitted, if the instructor or reader so desires.

Laplace transforms are covered in Chapter 7 and z-transforms are covered in Chapter 11. At many universities, one or both transforms are introduced prior to the signals and systems courses. Chapters 7 and 11 are written such that the material can be covered anywhere in the signals and systems course, or they can be omitted entirely, except for required references.

The more advanced material has been placed toward the end of the chapters wherever possible. Hence, this material may be omitted if desired. For example, Sections 3.7, 3.8, 4.6, 5.4, 7.9, 10.7, 12.6, and 12.7 could be omitted by instructors without loss of continuity in teaching. Further, Chapters 8 and 13 can be skipped if a professor does not wish to cover state-space material at the undergraduate level.

The material of this book is organized into two principal areas: continuous-time signals and systems, and discrete-time signals and systems. Some professors prefer to cover first one of these topics, followed by the second. Other professors prefer to cover continuous-time material and discrete-time material simultaneously. The authors have taken the first approach, with the continuous-time material covered in Chapters 2–8, and the discrete-time material covered in Chapters 9–13. The material on discrete-time concepts is essentially independent of the material on continuous-time concepts so that a professor or reader who desires to study the discrete-time material first could cover Chapters 9–11 and 13 before Chapters 2–8. The material may also be arranged such that basic continuous-time material and discrete-time material are intermixed. For example, Chapters 2 and 9 may be covered simultaneously and Chapters 3 and 10 may also be covered simultaneously.

In Chapter 1, we present a brief introduction to signals and systems, followed by short descriptions of several physical continuous-time and discrete-time systems. In addition, some of the signals that appear in these systems are described. Then a very brief introduction to MATLAB is given.

In Chapter 2, we present general material basic to continuous-time signals and systems; the same material for discrete-time signals and systems is presented in Chapter 9. However, as stated above, Chapter 9 can be covered before Chapter 2 or simultaneously with Chapter 2. Chapter 3 extends this basic material to continuous-time linear time-invariant systems, while Chapter 10 does the same for discrete-time linear time-invariant systems.

Presented in Chapters 4, 5, and 6 are the Fourier series and the Fourier transform for continuous-time signals and systems. The Laplace transform is then developed in Chapter 7. State variables for continuous-time systems are covered in Chapter 8; this development utilizes the Laplace transform.

The z-transform is developed in Chapter 11, with the discrete-time Fourier transform and the discrete Fourier transform presented in Chapter 12. However, Chapter 12 may be covered prior to Chapter 11. The development of the discrete-time Fourier transform and discrete Fourier transform in Chapter 12 assumes that the reader is familiar with the Fourier transform. State variables for discrete-time systems are given in Chapter 13. This material is independent of the state variables for continuous-time systems of Chapter 8.

In Appendix A, we give some useful integrals and trigonometric identities. In general, the table of integrals is used in the book, rather than taking the longer approach of integration by parts. Leibnitz's rule for the differentiation of an integral and L'Hôpital's rule for indeterminate forms are given in Appendix B and are referenced in the text where needed. Appendix C covers the closed forms for certain geometric series; this material is useful in discrete-time signals and systems. In Appendix D, we review complex numbers and introduce Euler's relation, in

Appendix E the solution of linear differential equations with constant coefficients, and in Appendix F partial-fraction expansions. Matrices are reviewed in Appendix G; this appendix is required for the state-variable coverage of Chapters 8 and 13. As each matrix operation is defined, MATLAB statements that perform the operation are given. Appendix H provides solutions to selected chapter problems so that students can check their work independently. Appendix I lists the references for the entire text, arranged by chapter.

This book may be covered in its entirety in two 3-semester-hour courses, or in quarter courses of approximately the equivalent of 6 semester hours. With the omission of appropriate material, the remaining parts of the book may be covered with fewer credits. For example, most of the material of Chapters 2, 3, 4, 5, 6, 8, 9, 10, 11, and 12 has been covered in one 4-semester-hour course. The students were already familiar with some linear-system analysis and the Laplace transform.

We wish to acknowledge the many colleagues and students at Auburn University, the University of Evansville, and the University of Washington who have contributed to the development of this book. In particular, the first author wishes to express thanks to Professors Charles M. Gross, Martial A. Honnell, and Charles L. Rogers of Auburn University for many stimulating discussions on the topics in this book, and to Professor Roger Webb, director of the School of Electrical Engineering at the Georgia Institute of Technology, for the opportunity to teach the signal and system courses at Georgia Tech. The second author wishes to thank Professors Dick Blandford and William Thayer for their encouragement and support for this effort, and Professor David Mitchell for his enthusiastic discussions of the subject matter. The third author wishes to thank the professors and many students in EE235 and EE341 at the University of Washington who contributed comments to this book and interactive website, in particular Professors Mari Ostendorf and Mani Soma, Eddy Ferré, Wai Shan Lau, Bee Ngo, Sanaz Namdar, Jessica Tsao, and Anna Margolis. We would like to thank the reviewers who provided invaluable comments and suggestions, including the following reviewers of the fifth edition: Sos Agaian, University of Texas, San Antonio, Tokunbo Ogunfunmi, Santa Clara University, and K. Sivakumar, Washington State University. The interactive website was developed under a grant from the Fund for the Improvement of Postsecondary Education (FIPSE), U.S. Department of Education.

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SIGNALS, SYSTEMS, AND TRANSFORMS

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1 INTRODUCTION

In this book, we consider the topics of signals and systems as related to engineering. These topics involve the *modeling* of physical signals by mathematical functions, the *modeling* of physical systems by mathematical equations, and the solutions of the equations when excited by the functions.

1.1 MODELING

Engineers must model two distinct physical phenomena. First, *physical systems* are modeled by *mathematical equations*. For systems that contain no sampling (*continuous-time,* or *analog, systems*), we prefer to use ordinary differential equations with constant coefficients; a wealth of information is available for the analysis and the design of systems of this type. Of course, the equation must accurately model the physical systems. An example of the model of a physical system is a linear electric-circuit model of Figure 1.1:

$$L\frac{di(t)}{dt} + Ri(t) + \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = v(t). \tag{1.1}$$

Another example is Newton's second law,

$$f(t) = M\frac{d^2x(t)}{dt^2}, \tag{1.2}$$

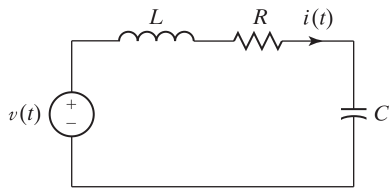

Figure 1.1 Example circuit.

where $f(t)$ is the force applied to the mass M and $x(t)$ is the resulting displacement of the mass.

A second physical phenomenon to be modeled is called *signals*. *Physical signals* are modeled by *mathematical functions*. One example of a physical signal is the voltage that is applied to the speaker in a radio. Another example is the temperature at a designated point in a particular room. This signal is a function of time because the temperature varies with time. We can express this temperature as

$$\text{temperature at a point} = \theta(t), \tag{1.3}$$

where $\theta(t)$ has the units of, for example, degrees Celsius.

Consider again Newton's second law. Equation (1.2) is the *model* of a physical system, and $f(t)$ and $x(t)$ are *models* of physical signals. Given the signal (function) $f(t)$, we solve the model (equation) (1.2) for the signal (function) $x(t)$. In analyzing physical systems, we apply mathematics to the *models* of systems and signals, not to the physical systems and signals. The usefulness of the results depends on the accuracy of the models.

In this book, we usually limit signals to having one independent variable. We choose this independent variable to be *time, t*, without loss of generality. Signals are divided into two natural categories. The first category to be considered is *continuous-time,* or simply, *continuous, signals*. A signal of this type is defined for all values of time. A continuous-time signal is also called an *analog signal*. A continuous-time signal is illustrated in Figure 1.2(a).

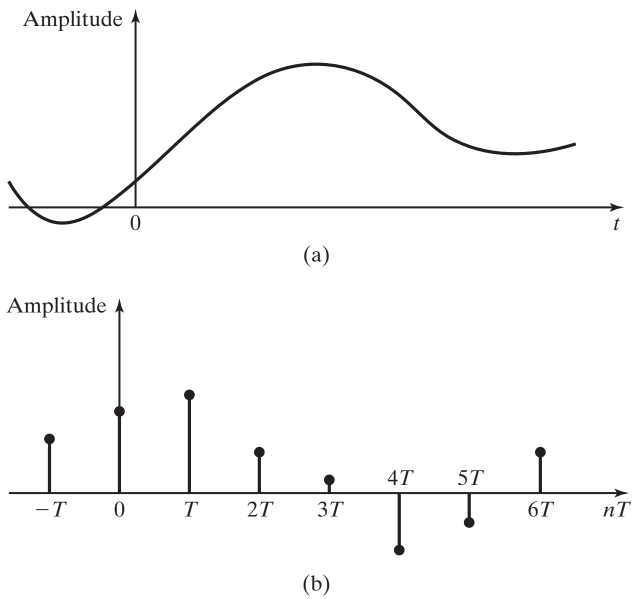

Figure 1.2 (a) Continuous-time signal; (b) discrete-time signal.

The second category for signals is *discrete-time,* or simply, *discrete, signals*. A discrete signal is defined at only certain instants of time. For example, suppose that a signal $f(t)$ is to be processed by a digital computer. [This operation is called *digital signal processing* (DSP).] Because a computer can operate only on numbers and not on a continuum, the continuous signal must be converted into a sequence of numbers by sampling. If a signal $f(t)$ is sampled every T seconds, the number sequence $f(nT), n = \ldots, -2, -1, 0, 1, 2, \ldots,$ is available to the computer. This sequence of numbers is called a *discrete-time signal*. Insofar as the computer is concerned, $f(nT)$ with n a noninteger does not exist (is not available). A discrete-time signal is illustrated in Figure 1.2(b).

We define a *continuous-time system* as one in which all signals are continuous time. We define a *discrete-time system* as one in which all signals are discrete time. Both continuous-time and discrete-time signals appear in some physical systems; we call these systems *hybrid systems,* or *sampled-data systems*. An example of a sampled-data system is an automatic aircraft-landing system, in which the control functions are implemented on a digital computer.

The mathematical analysis of physical systems can be represented as in Figure 1.3 [1]. We first develop mathematical models of the physical systems and signals involved. One procedure for finding the model of a physical system is to use the laws of physics, as, for example, in (1.1). Once a model is developed, the equations are solved for typical excitation functions. This solution is compared with the response of the physical system with the same excitation. If the two responses are approximately equal, we can then use the model in analysis and design. If not, we must improve the model.

Improving the mathematical model of a system usually involves making the models more complex and is not a simple step. Several iterations of the process illustrated in Figure 1.3 may be necessary before a model of adequate accuracy results. For some simple systems, the modeling may be completed in hours; for very complex systems, the modeling may take years. An example of a complex model is that of NASA's shuttle; this model relates the position and attitude of the shuttle to

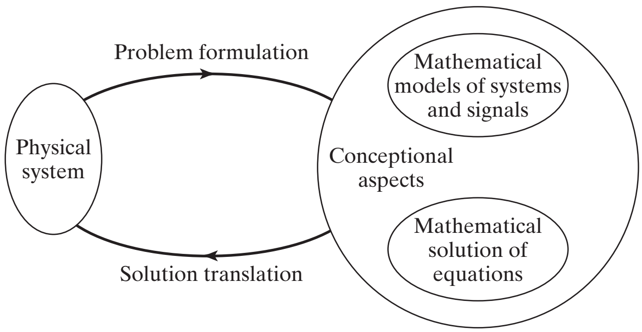

Figure 1.3 Mathematical solutions of physical problems.

the engine thrust, the wind, the positions of the control surfaces (e.g., the rudder), and so on. As an additional point, for complex models of this type, the equations can be solved only by computer.

This book contains two main topics: (1) continuous-time signals and systems and (2) discrete-time signals and systems. Chapters 2 through 8 cover continuous-time signals and systems, while Chapters 9 through 13 cover discrete-time signals and systems. The material may be covered in the order of the chapters, in which continuous-time topics and discrete-time topics are covered separately. Alternatively, the basic material of the two topics may be intermixed, with Chapters 2 and 9 covered simultaneously, followed by Chapters 3 and 10 covered simultaneously.

1.2 CONTINUOUS-TIME PHYSICAL SYSTEMS

In this section, we discuss several continuous-time physical systems. The descriptions are simplified; references are given that contain more complete descriptions. The systems described in this and the next section are used in examples throughout the remainder of the book.

We have already given the model of a rigid mass M in a frictionless environment,

$$f(t) = M\frac{d^2x(t)}{dt^2}, \quad \text{[eq(1.2)]}$$

where $f(t)$ is the force applied to the mass and $x(t)$ is the displacement of the mass that results from the force applied. This model is a *second-order linear differential equation with constant coefficients*.

Linearity is defined in Section 2.7. As we will see, an equation (or system) is *linear* if the principle of superposition applies. Otherwise, the equation is *nonlinear*.

Next we discuss several physical systems.

Electric Circuits

In this section, we give models for some electric-circuit elements [2]. We begin with the model for *resistance,* given by

$$v(t) = Ri(t), \quad \text{or} \quad i(t) = \frac{1}{R}v(t), \tag{1.4}$$

where the voltage $v(t)$ has the units of volts (V), the current $i(t)$ has the units of amperes (A), and the resistance R has the units of ohms (Ω). This model is represented by the standard circuit symbol given in Figure 1.4. The dashed lines in this figure indicate that the elements are parts of circuits. For example, the resistance must be a part of a circuit, or else $v(t)$ is identically zero.

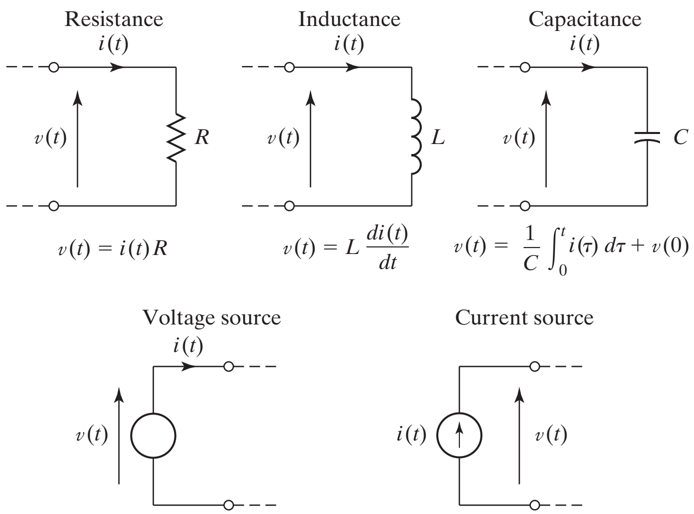

Figure 1.4 Electric-circuit elements. (From C. L. Phillips and R. D. Harbor, *Feedback Control Systems,* 3d ed., Prentice Hall, Upper Saddle River, NJ, 1995.)

The model for *inductance* is given by

$$v(t) = L\frac{di(t)}{dt}, \quad \text{or} \quad i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau, \tag{1.5}$$

where $v(t)$ and $i(t)$ are as defined earlier and L is the inductance in henrys. The model for *capacitance* is given by

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau, \quad \text{or} \quad i(t) = C\frac{dv(t)}{dt}, \tag{1.6}$$

where C is the capacitance in farads. The symbols for inductance and capacitance are also given in Figure 1.4.

For the ideal voltage source in Figure 1.4, the voltage at the terminals of the source is $v(t)$, independent of the circuit connected to these terminals. The current $i(t)$ that flows through the voltage source is determined by the circuit connected to the source. For the ideal current source, the current that flows through the current source is $i(t)$, independent of the circuit connected to the source. The voltage $v(t)$ that appears at the terminals of the current source is determined by the circuit connected to these terminals.

Consider now a circuit that is an interconnection of the elements shown in Figure 1.4. The circuit equations are written using the models given in the figure along with Kirchhoff's voltage and current laws. Kirchhoff's voltage law may be stated as follows:

> The algebraic sum of voltages around any closed loop in an electric circuit is zero.

Kirchhoff's current law may be stated as follows:

> The algebraic sum of currents into any junction in an electric circuit is zero.

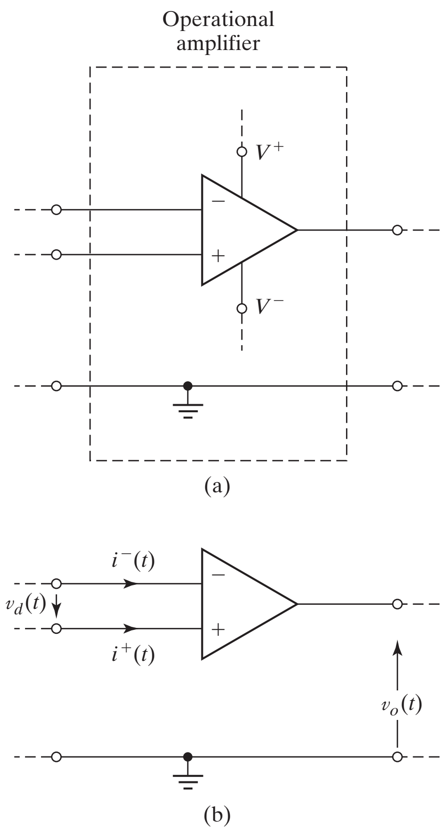

Figure 1.5 Operational amplifier.

Operational Amplifier Circuits

A device called an *operational amplifier* (or *op amp*) [3] is commonly used in circuits for processing analog electrical signals. We do not investigate the internal structure of this amplifier, but instead present only its terminal characteristics.

We denote an operational amplifier by the circuit symbol of Figure 1.5(a). The circles indicate amplifier terminals, and the dashed lines indicate connections external to the amplifier. The signal-input terminals are labeled with a minus sign for the *inverting input* and a plus sign for the *noninverting input*. The power-supply terminals are labeled V^+ for the positive dc voltage and V^- for the negative dc voltage. The op amp is normally shown as in Figure 1.5(b), with the power-supply terminals omitted. In this circuit, $v_d(t)$ is the input voltage to be amplified and the amplified voltage output is $v_o(t)$.

The operational amplifier is designed and constructed such that the input impedance is very high, resulting in the input currents $i^-(t)$ and $i^+(t)$ in Figure 1.5(b) being very small. Additionally, the amplifier gain [the ratio $v_o(t)/v_d(t)$] is very large (on the order of 10^5 or larger). This large gain results in a very small allowable input voltage if the amplifier is to operate in its linear range (not saturated).

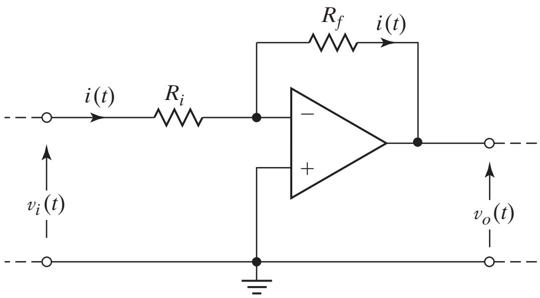

Figure 1.6 Practical voltage amplifier.

For this discussion, we assume that the amplifier is ideal, which is sufficiently accurate for most purposes. The ideal op amp has zero input currents $[i^-(t) = i^+(t) = 0]$. Additionally, the ideal amplifier operates in its linear range with infinite gain, resulting in the input voltage $v_d(t)$ being zero.

Because the op amp is a very high-gain device, feedback is usually added for stabilization. The feedback is connected from the output terminal to the inverting input terminal (the minus terminal). This connection results in negative, or stabilizing, feedback and tends to prevent saturation of the op amp.

An example of a practical op-amp circuit is given in Figure 1.6. In this circuit, $v_i(t)$ is the circuit input voltage and $v_o(t)$ the circuit output voltage. Because $v_d(t)$ in Figure 1.5(b) is assumed to be zero, the equation for the input loop in Figure 1.6 is given by

$$v_i(t) - i(t)R_i = 0 \Rightarrow i(t) = \frac{v_i(t)}{R_i}. \tag{1.7}$$

Also, because $i^-(t)$ in Figure 1.5(b) is zero, the current through R_f in Figure 1.6 is equal to that through R_i. The equation for the outer loop is then

$$v_i(t) - i(t)R_i - i(t)R_f - v_o(t) = 0.$$

Using (1.7), we express this equation as

$$v_i(t) - v_i(t) - \frac{v_i(t)}{R_i}R_f - v_o(t) = 0 \Rightarrow \frac{v_o(t)}{v_i(t)} = -\frac{R_f}{R_i}. \tag{1.8}$$

This circuit is then a *voltage amplifier*. The ratio R_f/R_i is a positive real number; hence, the amplifier voltage gain $v_o(t)/v_i(t)$ is a negative real number. The model (1.8) is a *linear algebraic equation*.

A second practical op-amp circuit is given in Figure 1.7. We use the preceding procedure to analyze this circuit. Because the input loop is unchanged, (1.7) applies, with $R_i = R$. The equation of the outer loop is given by

$$v_i(t) - i(t)R - \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau - v_o(t) = 0. \tag{1.9}$$

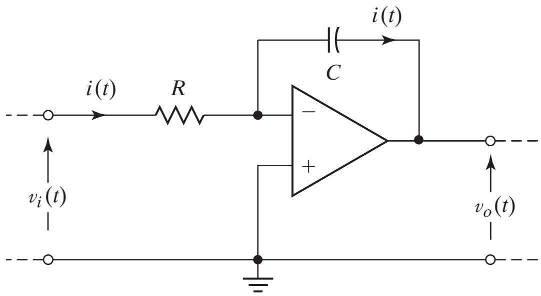

Figure 1.7 Integrating amplifier.

Substitution of (1.7) into (1.9) yields

$$v_i(t) - v_i(t) - \frac{1}{RC}\int_{-\infty}^{t} v_i(\tau)d\tau - v_o(t) = 0. \tag{1.10}$$

Thus, the equation describing this circuit is given by

$$v_0(t) = -\frac{1}{RC}\int_{-\infty}^{t} v_i(\tau)d\tau. \tag{1.11}$$

This circuit is called an *integrator* or an *integrating amplifier;* the output voltage is the integral of the input voltage multiplied by a negative constant $(-1/RC)$. This integrator is a commonly used circuit in analog signal processing and is used in several examples in this book.

If the positions of the resistance and the capacitance in Figure 1.7 are interchanged, the op-amp circuit of Figure 1.8 results. We state without proof that the equation of this circuit is given by

$$v_o(t) = -RC\frac{dv_i(t)}{dt}. \tag{1.12}$$

(The reader can show this by using the previous procedure.) This circuit is called a *differentiator,* or a *differentiating amplifier;* the output voltage is the derivative of the input voltage multiplied by a negative constant $(-RC)$. The differentiator has limited use in analog signal processing, because the derivative of a signal that changes

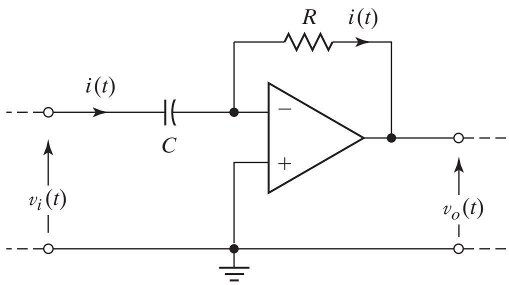

Figure 1.8 Differentiating amplifier.

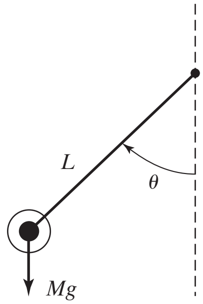

Figure 1.9 Simple pendulum.

rapidly is large. Hence, the differentiator amplifies any high-frequency noise in $v_i(t)$. However, some practical applications require the use of a differentiator. For these applications, some type of high-frequency filtering is usually required before the differentiation, to reduce high-frequency noise.

Simple Pendulum

We now consider a differential-equation model of the simple pendulum, which is illustrated in Figure 1.9. The angle of the pendulum is denoted as θ, the mass of the pendulum bob is M, and the length of the (weightless) arm from the axis of rotation to the center of the bob is L.

The force acting on the bob of the pendulum is then Mg, where g is the gravitational acceleration, as shown in Figure 1.9. From physics we recall the equation of motion of the simple pendulum:

$$ML\frac{d^2\theta(t)}{dt^2} = -Mg\sin\theta(t). \tag{1.13}$$

This model is a *second-order nonlinear differential equation;* the term $\sin\theta(t)$ is nonlinear. (Superposition does not apply.)

We have great difficulty in solving nonlinear differential equations; however, we can *linearize* (1.13). The power-series expansion for $\sin\theta$ is given (from Appendix D) by

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots. \tag{1.14}$$

For θ small, we can ignore all terms except the first one, resulting in $\sin\theta \approx \theta$ when θ is expressed in radians. The error in this approximation is less than 10 percent for $\theta = 45°$ ($\pi/4$ radians), is less than 1 percent for $\theta = 14°$ (0.244 radians), and decreases as θ becomes smaller. We then express the model of the pendulum as, from (1.13) and (1.14),

$$\frac{d^2\theta(t)}{dt^2} + \frac{g}{L}\theta(t) = 0 \tag{1.15}$$